SIMILAR SOLUTION OF PROBLEMS OF NONSTATIONARY GAS PERCOLATION THROUGH A BED WITH A NONLINEAR (TWO-TERM) RESISTANCE LAW

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Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 9, No. 6, pp. 159–160, 1968

In the nonstationary percolation regime a two-term resistance law can be written in the form [1]:

$$-\frac{\partial p}{\partial s}=\frac{\mu}{k}v+b\rho v^2\operatorname{sign}v. \tag{1}$$

Here, p is the pressure, v is the percolation rate, μ is the viscosity of the gas, k is the permeability, ρ is the density of the gas, s is a space coordinate, and b is the so-called turbulence coefficient.

Let us construct a similar solution of the one-dimensional problem of nonstationary percolation of a gas through a bed.

For simplicity, we select special initial and boundary conditions, i. e., we assume that at the initial instant the gas pressure in the bed is equal to zero $p(s, 0) = 0$ and that the bed is infinite. In accordance with [2], it must be assumed that in the given case there will be a finite rate of propagation of the disturbance.

We write the differential equation of nonstationary isothermal percolation for the case of a gas injected into a bed ($\operatorname{sign} v = 1$). Multiplying both sides of (1) by ρ, we obtain a quadratic equation in the product ρv; solving it we find

$$\rho v=-\frac{1}{2b}\left[a-\left(a^2-4b\rho\frac{\partial p}{\partial s}\right)^{1/2}\right].$$

Substituting the value of ρv into the continuity equation

$$\frac{1}{s^n}\frac{\partial}{\partial s}(s^n\rho v)=-m\frac{\partial \rho}{\partial t}\,.$$

we obtain

$$\frac{1}{S^n}\frac{\partial}{\partial s}\left\{s^n\left[a-\left(a^2-4b\frac{\rho_{am}}{p_{am}}p\frac{\partial p}{\partial s}\right)^{1/2}\right]\right\}=2mb\frac{\rho^\circ}{p^\circ}\frac{\partial p}{\partial t}\left(a=\frac{\mu}{k}\right). \tag{2}$$

Here, t is time, m is porosity, and $n = 0, 1, 2$ (plane motions, motions with axial and central symmetry).

It is required to find the solution of Eq. (2) with the conditions

$$\lim_{s\to 0}\frac{F(s)\,a}{2b}\left[-1+\left(1-\frac{4b}{a^2}\frac{\rho^\circ}{p^\circ}p\frac{\partial p}{\partial s}\right)^{1/2}\right]=At^{\beta},\qquad p(s,0)=0. \tag{3}$$

Here, F(s) is the cross-sectional area of the bed.

In the case of linear-parallel percolation F does not depend on s and is given by $F = 1\,h$; in cases of plane-radial and spherical-radial percolation $F = 2\pi hs$, $F = 2\pi s^2$, respectively.

Using dimensional analysis [3], we easily see that the solution of the problem is self-similar and has the form:

$$p=a\left(\frac{p^{\circ 2}t}{mb^2\rho^{\circ 2}}\right)^{1/3}f(\eta),\qquad \eta=\left(\frac{m^2b\rho^\circ}{p^\circ}\right)^{1/3}\frac{s}{t^{2/3}} \tag{4}$$

Substituting (4) into (2), we obtain

$$\eta\frac{d^2f^2}{d\eta^2}+n\left[-1+2\frac{df^2}{d\eta}+\left(1-2\frac{df^2}{d\eta}\right)^{1/2}\right]=\frac{2}{3}\eta\left(f-2\eta\frac{df}{d\eta}\right)\left(1-2\frac{df^2}{d\eta}\right)^{1/2}. \tag{5}$$

The boundary conditions for f have the form:

for n = 0 (β = 0)

$$\lim_{\eta \to 0} D_1 \left[-1 + \left(1 - 2\frac{df^2}{d\eta}\right)^{1/2}\right] = 1, \qquad f(\infty) = 0; \tag{6}$$

for $n = 1$ ($\beta = 2/3$)

$$\lim_{\eta \to 0} D_2 \eta \left[-1 + \left(1 - 2\frac{df^2}{d\eta}\right)^{1/2}\right] = 1, \qquad f(\infty) = 0; \tag{7}$$

for $n = 2$ ($\beta = 4/3$)

$$\lim_{\eta \to 0} D_3 \eta^2 \left[-1 + \left(1 - 2\frac{df^2}{d\eta}\right)^{1/2}\right] = 1, \qquad f(\infty) = 0, \tag{8}$$

$$D_1 = \frac{ha}{2bA}, \quad D_2 = \frac{\pi a h}{bA}\left(\frac{p^\circ}{m^2 b \rho^\circ}\right)^{1/3}, \quad D_3 = \frac{\pi a}{bA}\left(\frac{p^\circ}{m^2 b \rho^\circ}\right)^{2/3}.$$

Clearly, Eq. (5) can be solved only numerically. The numerical calculation can be carried out as follows. Values of f and $df^2/d\eta$ are assigned for $\eta = 1$, the Cauchy problem for Eq. (5) is solved by the Runge-Kutta method, the values of $df^2/d\eta$ are determined for $\eta = 0$, and then the values of D are found.

Here we confine ourselves to the case in which n = 0; the results are presented in the table.

Values of f

η	Numerical solution	approximate solution	
		(9) and (10)	(11)
0.00	0.902195	0.908295	0.910330
0.10	0.822601	0.825712	0.827949
0.20	0.738969	0.743169	0.745587
0.30	0.656495	0.660606	0.663174
0.40	0.574363	0.578014	0.580775
0.50	0.492670	0.495480	0.498397
0.60	0.416496	0.412916	0.416053
0.70	0.328353	0.330303	0.333617
0.80	0.246516	0.247790	0.249199
0.90	0.164077	0.165227	0.168819
1.00	0.081998	0.082462	0.083023

We obtain an approximate solution of the problem on the basis of the method proposed in [4] using various integral relations.

The approximate solution of the problem (with n = 0) using the method of integral relations is:

using the first integral relation (material balance equation)

$$f^2 = M_1 (1 - 2M_4^{-1}\eta + M_4^{-2}\eta^2); \tag{9}$$

using the second integral relation

$$f^2 = M_2 (1 - 2M_5^{-1}\eta + M_5^{-2}\eta); \tag{10}$$

using the third integral relation

$$f^2 = M_3 (1 - 2M_6^{-1}\eta + M_6^{-2}\eta^2),$$

$$M_1 = 4^{-2/3} D_1^{-4/3} (D_1^{-1} + 2)^{2/3}, \quad M_2 = 1/4 \{36/5 [1/3 (c^{3/2} - 1) - 1/2 D_1^{-1} (D^{-1} + 2)]\}^{2/3},$$
$$M_3 = 1/4 [36/7 D_1^{-1} (D_1^{-1} + 2)]^{2/3} \{-1 + 4D_1^2 (D_1^{-1} + 2)^{-2} [1/5 (1 - c^2 \sqrt{c}) - \tag{11}$$
$$- 1/3 c (1 - c\sqrt{c})]\}^{2/3}, \qquad c = 1 + D_1^{-1} (D_1^{-1} + 2),$$
$$M_4 = \frac{4M_1}{c-1}, \qquad M_5 = \frac{4M_2}{c-1}, \qquad M_6 = \frac{4M_3}{c-1}.$$

The results of calculations based on (9), (10), and (11) are also presented in the table.

As may be seen from the table, the approximate solutions almost coincide with the numerical solution.

Moreover, it follows from the table that other integral relations may also be used.

REFERENCES

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5 April 1967

Baku